

## 0.1 `gamma.mixed`: Mixed effects gamma regression

Use generalized multi-level linear regression if you have covariates that are grouped according to one or more classification factors. Gamma regression models a continuous, positive dependent variable.

While generally called multi-level models in the social sciences, this class of models is often referred to as mixed-effects models in the statistics literature and as hierarchical models in a Bayesian setting. This general class of models consists of linear models that are expressed as a function of both *fixed effects*, parameters corresponding to an entire population or certain repeatable levels of experimental factors, and *random effects*, parameters corresponding to individual experimental units drawn at random from a population.

### Syntax

```
z.out <- zelig(formula= y ~ x1 + x2 + tag(z1 + z2 | g),
               data=mydata, model="gamma.mixed")

z.out <- zelig(formula= list(mu=y ~ x1 + x2 + tag(z1, delta | g),
                           delta= ~ tag(w1 + w2 | g)), data=mydata, model="gamma.mixed")
```

### Inputs

`zelig()` takes the following arguments for mixed:

- **formula**: a two-sided linear formula object describing the systematic component of the model, with the response on the left of a `~` operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1 + ... + zn | g)` with `z1 + ... + zn` specifying the model for the random effects and `g` the grouping structure. Random intercept terms are included with the notation `tag(1 | g)`.

Alternatively, **formula** may be a list where the first entry, **mu**, is a two-sided linear formula object describing the systematic component of the model, with the response on the left of a `~` operator and the fixed effects terms, separated by `+` operators, on the right. Any random effects terms are included with the notation `tag(z1, delta | g)` with `z1` specifying the individual level model for the random effects, `g` the grouping structure and **delta** references the second equation in the list. The **delta** equation is one-sided linear formula object with the group level model for the random effects on the right side of a `~` operator. The model is specified with the notation `tag(w1 + ... + wn | g)` with `w1 + ... + wn` specifying the group level model and `g` the grouping structure.

### Additional Inputs

In addition, `zelig()` accepts the following additional arguments for model specification:

- **data**: An optional data frame containing the variables named in **formula**. By default, the variables are taken from the environment from which **zelig()** is called.
- **method**: a character string. The criterion is always the log-likelihood but this criterion does not have a closed form expression and must be approximated. The default approximation is "PQL" or penalized quasi-likelihood. Alternatives are "Laplace" or "AGQ" indicating the Laplacian and adaptive Gaussian quadrature approximations respectively.
- **na.action**: A function that indicates what should happen when the data contain NAs. The default action (**na.fail**) causes **zelig()** to print an error message and terminate if there are any incomplete observations.

Additionally, users may wish to refer to **lmer** in the package **lme4** for more information, including control parameters for the estimation algorithm and their defaults.

## Examples

### 1. Basic Example with First Differences

Attach sample data:

```
> data(coalition2)
```

Estimate model using optional arguments to specify approximation method for the log-likelihood, and the log link function for the Gamma family:

```
> z.out1 <- zelig(duration ~ invest + fract + polar + numst2 +
+   crisis + tag(1 | country), data = coalition2, model = "gamma.mixed",
+   method = "PQL", family = Gamma(link = log))
```

Summarize regression coefficients and estimated variance of random effects:

```
> summary(z.out1)
```

Set the baseline values (with the ruling coalition in the minority) and the alternative values (with the ruling coalition in the majority) for X:

```
> x.high <- setx(z.out1, numst2 = 1)
> x.low <- setx(z.out1, numst2 = 0)
```

Simulate expected values (**qi\$ev**) and first differences(**qi\$fd**):

```
> s.out1 <- sim(z.out1, x = x.high, x1 = x.low)
> summary(s.out1)
```

## Mixed effects gamma regression Model

Let  $Y_{ij}$  be the continuous, positive dependent variable, realized for observation  $j$  in group  $i$  as  $y_{ij}$ , for  $i = 1, \dots, M$ ,  $j = 1, \dots, n_i$ .

- The *stochastic component* is described by a Gamma model with scale parameter  $\alpha$ .

$$Y_{ij} \sim \text{Gamma}(y_{ij} | \lambda_{ij}, \alpha)$$

where

$$\text{Gamma}(y_{ij} | \lambda_{ij}, \alpha) = \frac{1}{\alpha^{\lambda_{ij}} \Gamma(\lambda_{ij})} y_{ij}^{\lambda_{ij}-1} \exp(-\{\frac{y_{ij}}{\alpha}\})$$

for  $\alpha, \lambda_{ij}, y_{ij} > 0$ .

- The  $q$ -dimensional vector of *random effects*,  $b_i$ , is restricted to be mean zero, and therefore is completely characterized by the variance covariance matrix  $\Psi$ , a  $(q \times q)$  symmetric positive semi-definite matrix.

$$b_i \sim \text{Normal}(0, \Psi)$$

- The *systematic component* is

$$\lambda_{ij} \equiv \frac{1}{X_{ij}\beta + Z_{ij}b_i}$$

where  $X_{ij}$  is the  $(n_i \times p \times M)$  array of known fixed effects explanatory variables,  $\beta$  is the  $p$ -dimensional vector of fixed effects coefficients,  $Z_{ij}$  is the  $(n_i \times q \times M)$  array of known random effects explanatory variables and  $b_i$  is the  $q$ -dimensional vector of random effects.

## Quantities of Interest

- The predicted values (`qi$pr`) are draws from the gamma distribution for each given set of parameters  $(\alpha, \lambda_{ij})$ , for

$$\lambda_{ij} = \frac{1}{X_{ij}\beta + Z_{ij}b_i}$$

given  $X_{ij}$  and  $Z_{ij}$  and simulations of  $\beta$  and  $b_i$  from their posterior distributions. The estimated variance covariance matrices are taken as correct and are themselves not simulated.

- The expected values (`qi$ev`) are simulations of the mean of the stochastic component given draws of  $\alpha$ ,  $\beta$  from their posteriors:

$$E(Y_{ij}|X_{ij}) = \alpha\lambda_{ij} = \frac{\alpha}{X_{ij}\beta}.$$

- The first difference (`qi$fd`) is given by the difference in expected values, conditional on  $X_{ij}$  and  $X'_{ij}$ , representing different values of the explanatory variables.

$$FD(Y_{ij}|X_{ij}, X'_{ij}) = E(Y_{ij}|X_{ij}) - E(Y_{ij}|X'_{ij})$$

- In conditional prediction models, the average predicted treatment effect (`qi$att.pr`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^M \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^M \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij} = 1) - \widehat{Y_{ij}(t_{ij} = 0)}\},$$

where  $t_{ij}$  is a binary explanatory variable defining the treatment ( $t_{ij} = 1$ ) and control ( $t_{ij} = 0$ ) groups. Variation in the simulations is due to uncertainty in simulating  $Y_{ij}(t_{ij} = 0)$ , the counterfactual predicted value of  $Y_{ij}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $t_{ij} = 0$ .

- In conditional prediction models, the average expected treatment effect (`qi$att.ev`) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^M \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^M \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij} = 1) - E[Y_{ij}(t_{ij} = 0)]\},$$

where  $t_{ij}$  is a binary explanatory variable defining the treatment ( $t_{ij} = 1$ ) and control ( $t_{ij} = 0$ ) groups. Variation in the simulations is due to uncertainty in simulating  $E[Y_{ij}(t_{ij} = 0)]$ , the counterfactual expected value of  $Y_{ij}$  for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to  $t_{ij} = 0$ .

## Output Values

The output of each `Zelig` command contains useful information which you may view. You may examine the available information in `z.out` by using `slotNames(z.out)`, see the fixed effect coefficients by using `summary(z.out)$coefs`, and a default summary of information through `summary(z.out)`. Other elements available through the operator are listed below.

- From the `zelig()` output stored in `summary(z.out)`, you may extract:
  - `fixef`: numeric vector containing the conditional estimates of the fixed effects.

- `ranef`: numeric vector containing the conditional modes of the random effects.
- `frame`: the model frame for the model.
- From the `sim()` output stored in `s.out`, you may extract quantities of interest stored in a data frame:
  - `qi$pr`: the simulated predicted values drawn from the distributions defined by the expected values.
  - `qi$ev`: the simulated expected values for the specified values of `x`.
  - `qi$fd`: the simulated first differences in the expected values for the values specified in `x` and `x1`.
  - `qi$ate.pr`: the simulated average predicted treatment effect for the treated from conditional prediction models.
  - `qi$ate.ev`: the simulated average expected treatment effect for the treated from conditional prediction models.

## How to Cite

To cite the *gamma.mixed* Zelig model:

Delia Bailey and Ferdinand Alimadhi. 2007. "gamma.mixed: Mixed effects gamma model" in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," <http://gking.harvard.edu/zelig>

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," <http://GKing.harvard.edu/zelig>.

Imai, Kosuke, Gary King, and Olivia Lau. (2008). "Toward A Common Framework for Statistical Analysis and Development." *Journal of Computational and Graphical Statistics*, Vol. 17, No. 4 (December), pp. 892-913.

## See also

Mixed effects gamma regression is part of `lme4` package by Douglas M. Bates (Bates 2007). For a detailed discussion of mixed-effects models, please see ?

# Bibliography

Bates, D. (2007), *lme4: Fit linear and generalized linear mixed-effects models*.