

# Figures 11 and 12 in Griffis and Stedinger (2007)

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At page 489 Griffis and Stedinger (2007) write:

To allow use of the LP3 distribution in future L-moment frequency analyses, simple expressions were developed for  $\tau_4$  as a function of  $\tau_3$  of the form

$$\tau_4 = a + b\tau_3 + c\tau_3^2 + d\tau_3^3$$

For given  $\tau_3$ , with the coefficients in Table 3, the approximations yield values of  $\tau_4$  accurate to within 0.008 over the range  $\tau_{3(min)} \leq \tau_3 \leq 0.9$ , wherein Table 3 specifies the minimum value  $\tau_{3(min)}$  for each value of  $\gamma_x$  with  $\sigma_x = 0$ .

The following function uses the above equation and Table 3 in Griffis and Stedinger (2007):

```
> tau4_LP3 <- function (tau3_LP3) {
+ # the approximations yield values of tau4 accurate to
+ # within 0.008 over the range tau3min <= tau3 <= 0.9
+ gammax <- c(-1.4, -1, -0.5, 0, 0.5, 1, 1.4)
+ table3abcd <- matrix(c(0.0602, -0.1673, 0.8010, 0.2897,
+ 0.0908, -0.1267, 0.7636, 0.2562,
+ 0.1166, -0.0439, 0.6247, 0.2939,
+ 0.1220, 0.0238, 0.6677, 0.1677,
+ 0.1152, 0.0639, 0.7486, 0.0645,
+ 0.1037, 0.0438, 0.9327, -0.0951,
+ 0.0776, 0.0762, 0.9771, -0.1394),
+ ncol=4, nrow=7, byrow=TRUE,
+ dimnames=list(paste("gammax=", gammax, sep=""), c("a","b","c","d")))
+
+ tau3s <- matrix(c(rep(1, length(tau3_LP3)), tau3_LP3, tau3_LP3^2, tau3_LP3^3),
+ nrow=4, ncol=length(tau3_LP3), byrow=TRUE,
+ dimnames=list(c("1","tau3","tau3^2","tau3^3"),
+ paste("tau3=", tau3_LP3, sep="")))
+
+ tau3min <- c(-0.2308, -0.1643, -0.0740, 0, 0.0774, 0.1701, 0.2366) %*%
+ t(rep(1, length(tau3_LP3)))
+ tau3max <- rep(0.9, 7) %*% t(rep(1, length(tau3_LP3)))
+ tau3s2 <- rep(1,7) %*% t(tau3_LP3)
+ keeptau4 <- (tau3s2 >= tau3min*0.9999) & (tau3s2 <= tau3max*1.0001)
+
+ tau4s <- table3abcd %*% tau3s
+ tau4s[!keeptau4] <- NA
+
+ return(tau4s)
+ }
```

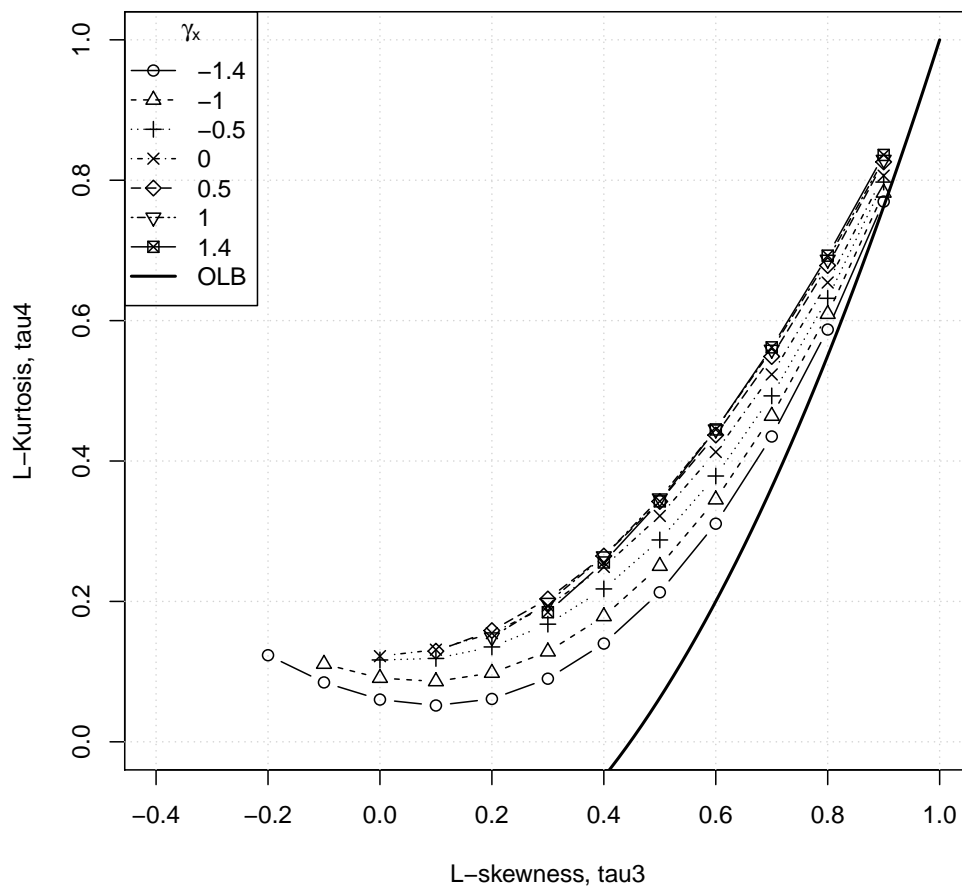
Figure 11 is then given by:

```
> gammax <- c(-1.4, -1, -0.5, 0, 0.5, 1, 1.4)
> tau3 <- seq(-0.2, 1, by=0.1)
> tau4 <- tau4_LP3(tau3)
> plot(c(-0.4, 1), c(0, 1), type="n", xlab="L-skewness, tau3", ylab="L-Kurtosis, tau4")
> grid()
> for (i in 1:7) {
```

```

+ lines(tau3, tau4[i,], type="b", lty=i, pch=i)
+ }
> legend("topleft", legend=c(gammax, "OLB"), title=expression(gamma[x]),
+       pch=c(1:7,NA), lty=c(1:7,1), lwd=c(rep(1,7),2))
> curve((5*x^2 - 1)/4, add=TRUE, lwd=2)

```



**Fig. 11.** L-moment ratio diagram for the LP3 distribution as a function of log space skew (OLB represents the overall theoretical lower bound of the  $\tau_3 - \tau_4$  space).

For Figure 12 load the library

```

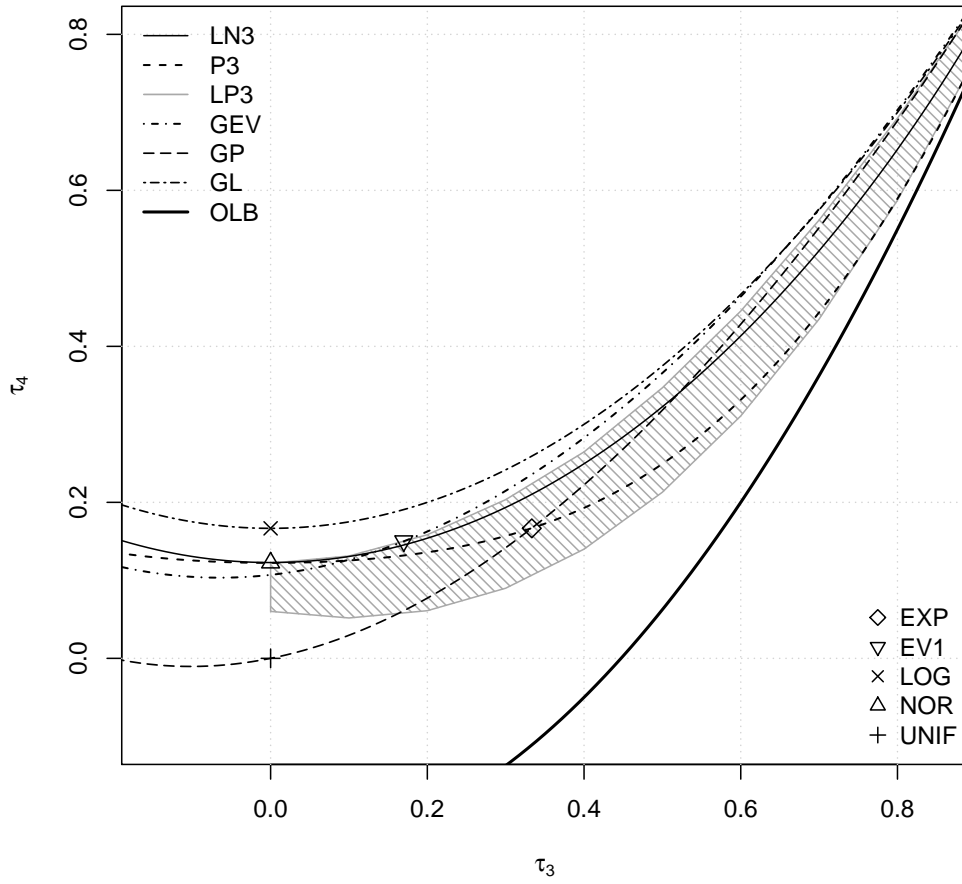
> library(nsRFA)
and do the following:
> t3b=-0.2; t3t=0.9; t4b=-0.1; t4t=0.8
> plot(c(t3b+0.05, t3t-0.05), c(t4b, t4t), type="n",
+      xlab=expression(tau[3]), ylab=expression(tau[4]), main="")
> grid()
> tipi <- c(1,2,1,4,5,6)
> spessori <- c(1,1.3,1.1,1.3,1.1,1.1)
> colori <- c(1,1,"darkgrey",1,1,1)
> tau3 <- seq(0, 0.9, by=0.1)
> tau4 <- tau4_LP3(tau3)
> tau4r <- apply(tau4, 2, range, na.rm=TRUE)

```

```

> polygon(x=c(tau3, rev(tau3)), y=c(tau4r[1,], rev(tau4r[2,])),
+         density=20, col="darkgrey", border="darkgrey", angle=-45)
> GPA <- function(x) 0.20196*x + 0.95924*x^2 - 0.20096*x^3 + 0.04061*x^4
> curve(GPA, t3b, t3t, add=TRUE, lty=tipi[5], lwd=spessori[5])
> GEV <- function(x) {
+   0.10701 + 0.1109*x + 0.84838*x^2 - 0.06669*x^3 +
+   0.00567*x^4 - 0.04208*x^5 + 0.03763*x^6
+ }
> curve(GEV, t3b, t3t, add=TRUE, lty=tipi[4], lwd=spessori[4])
> GLO <- function(x) 0.16667 + 0.83333*x^2
> curve(GLO, t3b, t3t, add=TRUE, lty=tipi[6], lwd=spessori[6])
> LN3 <- function(x) 0.12282 + 0.77518*x^2 + 0.12279*x^4 - 0.13638*x^6 + 0.11368*x^8
> curve(LN3, t3b, t3t, add=TRUE, lty=tipi[1], lwd=spessori[1])
> PE3 <- function(x) 0.1224 + 0.30115*x^2 + 0.95812*x^4 - 0.57488*x^6 + 0.19383*x^8
> curve(PE3, t3b, t3t, add=TRUE, lty=tipi[2], lwd=spessori[2])
> points(0, 0, pch=3, cex=1.2)
> points(0, 0.1226, pch=2, cex=1.2)
> points(1/3, 1/6, pch=5, cex=1.2)
> points(0.1699, 0.1504, pch=6, cex=1.2)
> points(0, 1/6, pch=4, cex=1.2)
> curve((5*x^2 - 1)/4, t3b, t3t, add=TRUE, lwd=2)
> legend("bottomright", c("EXP", "EV1", "LOG", "NOR", "UNIF"),
+       pch=c(5, 6, 4, 2, 3), bty="n")
> legend("topleft", legend=c("LN3", "P3", "LP3", "GEV", "GP", "GL", "OLB"),
+       lty=c(tipi,1), lwd=c(spessori,2), col=c(colori,1), bty="n")

```



**Fig. 12.** L-moment ratio diagram including GLO, GEV, LN, generalized Pareto GPA , P3, Gumbel, normal, and LP3 (light gray region represents LP3 distribution with  $|\gamma_x| \leq 1.414$ ; I have not plotted the dark gray region with restricted values of  $\sigma_x$ ; I added some other distributions.

## References

Griffis, V.W., and Stedinger, J.R. (2007). Log-Pearson Type 3 distribution and its application in Flood Frequency Analysis. 1: Distribution characteristics. *Journal of Hydrologic Engineering*, 12(5):482–491, DOI:10.1061/(ASCE)1084-0699(2007)12:5(482).