

# Package ‘abcrlda’

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**Type** Package

**Title** Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis

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## Description

Offers methods to perform asymptotically bias-corrected regularized linear discriminant analysis (ABC\_RLDA) for cost-sensitive binary classification. The bias-correction is an estimate of the bias term added to regularized discriminant analysis (RLDA) that minimizes the overall risk. The default magnitude of misclassification costs are equal and set to 0.5; however, the package also offers the options to set them to some predetermined values or, alternatively, take them as hyperparameters to tune.  
A. Zollanvari, M. Abdirash, A. Dadlani and B. Abibullaev (2019) <[doi:10.1109/LSP.2019.2918485](https://doi.org/10.1109/LSP.2019.2918485)>.

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abcrlda	<i>Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis for Cost-Sensitive Binary Classification</i>
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## Description

Constructs Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis.

## Usage

```
abcrlda(x, y, gamma = 1, cost = c(0.5, 0.5), bias_correction = TRUE)
```

## Arguments

x	Input matrix or data.frame of dimension nobs x nvars; each row is an feature vector.
y	A numeric vector or factor of class labels. Factor should have either two levels or be a vector with two distinct values. If y is presented as a vector, it will be coerced into a factor. Length of y has to correspond to number of samples in x.
gamma	Regularization parameter $\gamma$ in the ABC-RLDA discriminant function given by:

$$W_{ABC}^{RLDA} = \gamma \left( x - \frac{\bar{x}_0 + \bar{x}_1}{2} \right)^T H (\bar{x}_0 - \bar{x}_1) - \log \left( \frac{C_{01}}{C_{10}} \right) + \hat{\omega}_{opt}$$

$$H = (I_p + \gamma \hat{\Sigma})^{-1}$$

Formulas and derivations for parameters used in above equation can be found in the article under reference section.

cost	Parameter that controls the overall misclassification costs. This is a vector of length 1 or 2 where the first value is $C_{10}$ (represents the cost of assigning label 1 when the true label is 0) and the second value, if provided, is $C_{01}$ (represents the cost of assigning label 0 when the true label is 1). The default setting is c(0.5, 0.5), so both classes have equal misclassification costs If a single value is provided, it should be normalized to lie between 0 and 1 (but not including 0 or 1). This value will be assigned to $C_{10}$ while $C_{01}$ will be equal to $(1 - C_{10})$ .
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bias\_correction

Takes in a boolean value. If bias\_correction is TRUE, then asymptotic bias correction will be performed. Otherwise, (if bias\_correction is FALSE) asymptotic bias correction will not be performed and the ABCRLDA is the classical RLDA. The default is TRUE.

### Value

An object of class "abcrlda" is returned which can be used for class prediction (see predict()).

a                   Coefficient vector of a discriminant hyperplane:  $W(\mathbf{x}) = \mathbf{a}'\mathbf{x} + m$ .

m                   Intercept of discriminant hyperplane:  $W(\mathbf{x}) = \mathbf{a}'\mathbf{x} + m$ .

cost                Vector of cost values that are used to construct ABC-RLDA.

ncost              Normalized cost such that  $C_{10} + C_{01} = 1$ .

gamma              Regularization parameter value used in ABC\_RLDA discriminant function.

lev                 Levels corresponding to the labels in y.

### Reference

A. Zollanvari, M. Abdirash, A. Dadlani and B. Abibullaev, "Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis for Cost-Sensitive Binary Classification," in IEEE Signal Processing Letters, vol. 26, no. 9, pp. 1300-1304, Sept. 2019. doi: 10.1109/LSP.2019.2918485 URL: <https://ieeexplore.ieee.org/document/8720003>

### See Also

Other functions in the package: [cross\\_validation\(\)](#), [da\\_risk\\_estimator\(\)](#), [grid\\_search\(\)](#), [predict.abcrlda\(\)](#), [risk\\_calculate\(\)](#)

### Examples

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                        iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                                iris[, ncol(iris)] == "versicolor"), 5])
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = 0.75)
a <- predict(model, train_data)
# same params but more explicit
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = c(0.75, 0.25))
b <- predict(model, train_data)
# same class costs ratio
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = c(3, 1))
c <- predict(model, train_data)
# all this model will give the same predictions
all(a == b & a == c & b == c)
#' [1] TRUE
```

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cross\_validation      *Cross Validation for separate sampling adjusted for cost.*

---

### Description

This function implements Cross Validation for separate sampling adjusted for cost.

### Usage

```
cross_validation(
  x,
  y,
  gamma = 1,
  cost = c(0.5, 0.5),
  nfolds = 10,
  bias_correction = TRUE
)
```

### Arguments

- x**                    Input matrix or data.frame of dimension nobs x nvars; each row is an feature vector.
- y**                    A numeric vector or factor of class labels. Factor should have either two levels or be a vector with two distinct values. If y is presented as a vector, it will be coerced into a factor. Length of y has to correspond to number of samples in x.
- gamma**                Regularization parameter  $\gamma$  in the ABC-RLDA discriminant function given by:

$$W_{ABC}^{RLDA} = \gamma \left( x - \frac{\bar{x}_0 + \bar{x}_1}{2} \right)^T H (\bar{x}_0 - \bar{x}_1) - \log \left( \frac{C_{01}}{C_{10}} \right) + \hat{\omega}_{opt}$$

$$H = (I_p + \gamma \hat{\Sigma})^{-1}$$

Formulas and derivations for parameters used in above equation can be found in the article under reference section.

- cost**                Parameter that controls the overall misclassification costs. This is a vector of length 1 or 2 where the first value is  $C_{10}$  (represents the cost of assigning label 1 when the true label is 0) and the second value, if provided, is  $C_{01}$  (represents the cost of assigning label 0 when the true label is 1). The default setting is c(0.5, 0.5), so both classes have equal misclassification costs
- If a single value is provided, it should be normalized to lie between 0 and 1 (but not including 0 or 1). This value will be assigned to  $C_{10}$  while  $C_{01}$  will be equal to  $(1 - C_{10})$ .
- nfolds**              Number of folds to use with cross-validation. Default is 10. In case of imbalanced data, nfolds should not be greater than the number of observations in smaller class.

bias\_correction

Takes in a boolean value. If bias\_correction is TRUE, then asymptotic bias correction will be performed. Otherwise, (if bias\_correction is FALSE) asymptotic bias correction will not be performed and the ABCRLDA is the classical RLDA. The default is TRUE.

### Value

Returns list of parameters.

risk\_cross Returns risk estimation where  $\mathfrak{R} = \varepsilon_0 * C_{10} + \varepsilon_1 * C_{01}$   
 e\_0 Error estimate for class 0.  
 e\_1 Error estimate for class 1.

### Reference

Braga-Neto, Ulisses & Zollanvari, Amin & Dougherty, Edward. (2014). Cross-Validation Under Separate Sampling: Strong Bias and How to Correct It. *Bioinformatics* (Oxford, England). 30. 10.1093/bioinformatics/btu527. URL: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4296143/pdf/btu527.pdf>

### See Also

Other functions in the package: [abcrllda\(\)](#), [da\\_risk\\_estimator\(\)](#), [grid\\_search\(\)](#), [predict.abcrllda\(\)](#), [risk\\_calculate\(\)](#)

### Examples

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                        iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                                iris[, ncol(iris)] == "versicolor"), 5])
cross_validation(train_data, train_label, gamma = 10)
```

---

da\_risk\_estimator      *Double Asymptotic Risk Estimator*

---

### Description

This function implements the generalized (double asymptotic) consistent estimator of risk.

### Usage

```
da_risk_estimator(object)
```

### Arguments

object      An object of class "abcrllda".

**Value**

Calculates risk based on estimated class error rates and misclassification costs

$$\mathfrak{R} = \varepsilon_0 * C_{10} + \varepsilon_1 * C_{01}$$

**Reference**

A. Zollanvari, M. Abdirash, A. Dadlani and B. Abibullaev, "Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis for Cost-Sensitive Binary Classification," in IEEE Signal Processing Letters, vol. 26, no. 9, pp. 1300-1304, Sept. 2019. doi: 10.1109/LSP.2019.2918485  
URL: <https://ieeexplore.ieee.org/document/8720003>

**See Also**

Other functions in the package: [abcrlda\(\)](#), [cross\\_validation\(\)](#), [grid\\_search\(\)](#), [predict.abcrlda\(\)](#), [risk\\_calculate\(\)](#)

**Examples**

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                        iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                                iris[, ncol(iris)] == "versicolor"), 5])
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = 0.75)
da_risk_estimator(model)
```

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grid\_search

*Grid Search*


---

**Description**

Performs grid search to estimate the optimal hyperparameters (gamma and cost) within specified space based on double asymptotic risk estimation or cross validation. Double asymptotic risk estimation is more efficient to compute because it uses closed form for risk estimation. For further details, refer to the article in the reference section.

$$\mathfrak{R} = \varepsilon_0 * C_{10} + \varepsilon_1 * C_{01}$$

$$\varepsilon_i = \Phi\left(\frac{(-1)^{i+1}(\hat{G}_i + \hat{\omega}_{opt}/\gamma)}{\sqrt{\hat{D}}}\right)$$

Separate sampling cross-validation (see cross-validation function) was adapted to work with cost-based risk estimation.

**Usage**

```

grid_search(
  x,
  y,
  range_gamma,
  range_cost,
  method = "estimator",
  nfolds = 10,
  bias_correction = TRUE
)

```

**Arguments**

x	Input matrix or data.frame of dimension nobs x nvars; each row is an feature vector.
y	A numeric vector or factor of class labels. Factor should have either two levels or be a vector with two distinct values. If y is presented as a vector, it will be coerced into a factor. Length of y has to correspond to number of samples in x.
range_gamma	Vector of gamma values to check.
range_cost	nobs x 1 vector (values should be between 0 and 1) or nobs x 2 matrix (each row is cost pair value $c(C_{10}, C_{01})$ ) of cost values to check.
method	Selects method to evaluate risk. "estimator" and "cross".
nfolds	Number of folds to use with cross-validation. Default is 10. In case of imbalanced data, nfolds should not be greater than the number of observations in smaller class.
bias_correction	Takes in a boolean value. If bias_correction is TRUE, then asymptotic bias correction will be performed. Otherwise, (if bias_correction is FALSE) asymptotic bias correction will not be performed and the ABCRLDA is the classical RLDA. The default is TRUE.

**Value**

List of estimated parameters.

cost	Cost value for which risk estimates are lowest during the search.
gamma	Gamma regularization parameter for which risk estimates are lowest during the search.
risk	Lowest risk value estimated during grid search.

**Reference**

A. Zollanvari, M. Abdirash, A. Dadlani and B. Abibullaev, "Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis for Cost-Sensitive Binary Classification," in IEEE Signal Processing Letters, vol. 26, no. 9, pp. 1300-1304, Sept. 2019. doi: 10.1109/LSP.2019.2918485 URL: <https://ieeexplore.ieee.org/document/8720003>

Braga-Neto, Ulisses & Zollanvari, Amin & Dougherty, Edward. (2014). Cross-Validation Under Separate Sampling: Strong Bias and How to Correct It. *Bioinformatics* (Oxford, England). 30. 10.1093/bioinformatics/btu527. URL: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4296143/pdf/btu527.pdf>

### See Also

Other functions in the package: [abcrlda\(\)](#), [cross\\_validation\(\)](#), [da\\_risk\\_estimator\(\)](#), [predict.abcrlda\(\)](#), [risk\\_calculate\(\)](#)

### Examples

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                        iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                                iris[, ncol(iris)] == "versicolor"), 5])
cost_range <- seq(0.1, 0.9, by = 0.2)
gamma_range <- c(0.1, 1, 10, 100, 1000)

gs <- grid_search(train_data, train_label,
                 range_gamma = gamma_range,
                 range_cost = cost_range,
                 method = "estimator")
model <- abcrlda(train_data, train_label,
                gamma = gs$gamma, cost = gs$cost)
predict(model, train_data)

cost_range <- matrix(1:10, ncol = 2)
gamma_range <- c(0.1, 1, 10, 100, 1000)

gs <- grid_search(train_data, train_label,
                 range_gamma = gamma_range,
                 range_cost = cost_range,
                 method = "cross")
model <- abcrlda(train_data, train_label,
                gamma = gs$gamma, cost = gs$cost)
predict(model, train_data)
```

---

predict.abcrlda

*Class Prediction for abcrlda objects*

---

### Description

Classifies observations based on a given abcrlda object.

### Usage

```
## S3 method for class 'abcrlda'
predict(object, newx, ...)
```

**Arguments**

object            An object of class "abcrlda".  
newx             Matrix of new values for x at which predictions are to be made.  
...                Argument used by generic function predict(object, x, ...).

**Value**

Returns factor vector with predictions (i.e., assigned labels) for each observation. Factor levels are inherited from the object variable.

**Reference**

A. Zollanvari, M. Abdirash, A. Dadlani and B. Abibullaev, "Asymptotically Bias-Corrected Regularized Linear Discriminant Analysis for Cost-Sensitive Binary Classification," in IEEE Signal Processing Letters, vol. 26, no. 9, pp. 1300-1304, Sept. 2019. doi: 10.1109/LSP.2019.2918485  
URL: <https://ieeexplore.ieee.org/document/8720003>

**See Also**

Other functions in the package: [abcrlda\(\)](#), [cross\\_validation\(\)](#), [da\\_risk\\_estimator\(\)](#), [grid\\_search\(\)](#), [risk\\_calculate\(\)](#)

**Examples**

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                        iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                                iris[, ncol(iris)] == "versicolor"), 5])
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = 0.75)
a <- predict(model, train_data)
# same params but more explicit
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = c(0.75, 0.25))
b <- predict(model, train_data)
# same class costs ratio
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = c(3, 1))
c <- predict(model, train_data)
# all this model will give the same predictions
all(a == b & a == c & b == c)
#' [1] TRUE
```

---

risk\_calculate

*Risk Calculate*

---

**Description**

Estimates risk and error by applying a constructed classifier (an object of class abcrlda) to a given set of observations.

**Usage**

```
risk_calculate(object, x_true, y_true)
```

**Arguments**

object	An object of class "abcrlda".
x_true	Matrix of values for x for which true class labels are known.
y_true	A numeric vector or factor of true class labels. Factor should have either two levels or be a vector with two distinct values. If y_true is presented as a vector, it will be coerced into a factor. Length of y_true has to correspond to number of samples in x_test.

**Value**

A list of parameters where

actual_err0	Error rate for class 0.
actual_err1	Error rate for class 1.
actual_errTotal	Error rate overall.
actual_normrisk	Risk value normalized to be between 0 and 1.
actual_risk	Risk value without normalization.

**See Also**

Other functions in the package: [abcrlda\(\)](#), [cross\\_validation\(\)](#), [da\\_risk\\_estimator\(\)](#), [grid\\_search\(\)](#), [predict.abcrlda\(\)](#)

**Examples**

```
data(iris)
train_data <- iris[which(iris[, ncol(iris)] == "virginica" |
                       iris[, ncol(iris)] == "versicolor"), 1:4]
train_label <- factor(iris[which(iris[, ncol(iris)] == "virginica" |
                               iris[, ncol(iris)] == "versicolor"), 5])
model <- abcrlda(train_data, train_label, gamma = 0.5, cost = 0.75)
risk_calculate(model, train_data, train_label)
```

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